

# Decay and decoupling of heavy right-handed Majorana neutrinos in the L–R model

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**Abstract.** Heavy right-handed neutrinos are of current interest. The interactions and decay of such neutrinos determine their decoupling epoch during the evolution of the universe. This in turn affects various observable features like the energy density, nucleosynthesis, CMBR spectrum, galaxy formation and baryogenesis. Here, we consider reduction of right-handed electron-type Majorana neutrinos, in the left–right symmetric model, by the  $W_R^+ W_R^-$  channel and the channel originating from an anomaly, involving the  $SU(2)_R$  gauge group, as well as decay of such neutrinos. We study the reduction of these neutrinos for different ranges of left–right model parameters, and find that, if the neutrino mass exceeds the right-handed gauge boson mass, then the neutrinos never decouple for realistic values of the parameters, but, rather, decay in equilibrium. Because there is no out-of-equilibrium decay, no mass bounds can be set for the neutrinos.

## 1 Introduction

If a see-saw mechanism [1] is to account for left-handed electron neutrino masses  $\nu$  sufficiently small to be consistent with current ideas on neutrino oscillations, right-handed  $N$  neutrinos with mass  $M$  in the TeV scale come into the picture [2, 3]. High  $N$  masses in the range 1–20 TeV, and even higher masses, have been considered in studies of leptogenesis–baryogenesis [2, 4, 5] and  $e^- e^-$  collisions [3]. While some studies [5] consider  $M$  smaller than  $M_W$ , the right-handed  $W_R$  boson mass, others specifically use  $N$  masses greater than the  $W_R$  mass [3, 6]. As cosmological and laboratory lower bounds for  $Z'$ ,  $W_R$  masses are of the order of 0.5 TeV, 1.6–3.2 TeV, respectively [7], there is no reason not to consider  $N$  masses greater than the  $Z'$ ,  $W_R$  masses. Cosmological mass bounds for  $M$ , with  $M \gg M_W$ , have been considered in [8–12].

In a recent work [12],  $(B + L)$ -violation from an anomaly, involving the  $SU(2)_R$  gauge group [2, 13, 14], was considered as a generation/reduction channel for  $N$  neutrinos satisfying  $M > M_W$ . It was found that this anomalous channel played a role in the decoupling of such neutrinos, at least as important as the  $N\bar{N} \rightarrow W_R^+ W_R^-$  channel (each of these channels was found to be more important than the  $N\bar{N} \rightarrow F\bar{F}$  channel,  $F$  representing a relevant fermion). Matrix elements for  $N\bar{N} \rightarrow F\bar{F}$  and  $N\bar{N} \rightarrow W_R^+ W_R^-$  were calculated in [12] from the left–right symmetric extension [15, 16] of the standard model, as an illustration.

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In the above work [12], the  $N$  neutrinos were assumed to be stable, for simplicity. If, however, the left–right symmetric model (L–R model, hereafter) is to be taken as a serious working basis,  $N$  neutrinos cannot be considered to be stable. Decays involving  $\nu$ – $N$  mixing,  $W_L$ – $W_R$  mixing, generation mixing and  $CP$ -violating decays have been extensively studied. The last scenario has been widely used in generating lepton and hence baryon number from the decay of Majorana-type  $N$  neutrinos [4, 5, 17–19]. If the  $N$  mass is considered to be greater than the  $W_R$  mass, then these decay channels will be marginalised by the channel  $N \rightarrow W_R^+ + e^-$ . Such a fast decay should have important effects on decoupling. It is this which is studied in detail in the present paper. The effect on leptogenesis has already received considerable attention [20].

The effect of the decay of massive neutrinos on energy density, nucleosynthesis, the cosmic microwave background radiation (CMBR), galaxy formation and stellar evolution has been well studied [21, 22]. In these studies, the decay time was taken to be large (typically, larger than 100–200 s), and the effect of decay at the crucial epochs was followed. The decay time was chosen greater than the decoupling time, i.e., it was assumed that the neutrinos first decoupled, and, then, their cosmological and astrophysical effects were felt, as a result of subsequent out-of-equilibrium decay [22]. If a fast decay like  $N \rightarrow W_R^+ + e^-$  is considered, the relationship of decoupling and decay may not be like this, and the two have been considered together, in the present paper, in one Boltzmann equation.

The plan of the paper is as follows: Sect. 1 is the introduction. In Sect. 2, the thermally averaged  $N\bar{N} \rightarrow W_R^+ W_R^-$  annihilation cross-section times relative velocity

and the thermally averaged decay rate are calculated in the L–R model, assuming pure Majorana neutrinos. The anomalous rate per unit volume, used in [12], is slightly modified to accommodate pure Majorana neutrinos. In Sect. 3, the Boltzmann equation is written down and treated approximately to obtain a decoupling criterion. The possibility of decoupling is investigated numerically. Section 4 discusses the conclusions.

## 2 Annihilation, decay, and anomalous reduction of right-handed neutrinos: thermal averages in the L–R model

### 2.1 Summary of relevant features of the L–R model

In the L–R model, there are two doublets,  $(\nu, e_L)$  and  $(N, e_R)$ , belonging to the representations  $(1/2, 0, -1)$  and  $(0, 1/2, -1)$ , respectively, of  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , where the quantum numbers refer to the values of  $T_L, T_R$ , and  $B-L$  respectively. To simplify the issues, only one generation is considered (the lightest), and  $N-\nu$ ,  $W_L-W_R$ , and  $Z-Z'$  mixings are neglected.

The symmetry-breaking from  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y$  is achieved by means of a scalar triplet ( $T_L = 0, T_R = 1$  and  $B-L = 2$ )  $\Delta \equiv (\Delta^{++}, \Delta^+, \Delta^0)$ , putting  $\langle \Delta^0 \rangle = v_R/\sqrt{2}$  (chosen real). The right-handed gauge boson becomes massive due to the piece of the Lagrangian

$$L_{\Delta W+W^-} = D^\mu \Delta^\dagger D_\mu \Delta, \quad (1)$$

with  $D_\mu = \partial_\mu + ig\mathbf{T} \cdot \mathbf{W}_\mu + i(g'/2)B_\mu$ . From now on, the subscript R is dropped except when essential. The  $T_i$  form a  $3 \times 3$  representation of the  $SU(2)$  generators in a spherical basis.  $Z'_\mu$  is defined by

$$Z'_\mu = (\sqrt{\cos 2\theta}/\cos \theta)W_3^\mu - \tan \theta B^\mu. \quad (2)$$

Equation (1) gives a  $W$  mass  $M_W^2 = (1/2)g^2 v_R^2$  and an interaction

$$L_{HW+W^-} = \sqrt{2}gM_W W_\mu^+ W^{\mu-} H, \quad (3)$$

where  $H(x)/\sqrt{2} = \text{Re}(\Delta^0(x)) - v_R/\sqrt{2}$ , the displaced neutral Higgs field.

Neglecting  $Z-Z'$  mixing, one gets the  $Z'$  neutral current piece

$$L_{N\bar{N}}^{Z'} = (g/2\sqrt{\cos 2\theta}) \cos \theta \bar{N} \gamma^\mu P_R N Z'_\mu. \quad (4)$$

$\theta$  is the weak mixing angle. The required charged current piece of the Lagrangian is

$$L_{N_e}^W = (g/\sqrt{2})(\bar{N} \gamma^\mu P_R e W_\mu + \text{h.c.}). \quad (5)$$

The  $Z', W^+, W^-$  interaction piece is

$$\begin{aligned} L_{W+W^-}^{Z'} &= -ig(\sqrt{\cos 2\theta}/\cos \theta)[Z'^\mu(\partial_\mu W_\nu^+ W^{\nu-} - \partial_\nu W_\mu^+ W^{\nu-} \\ &\quad - \partial_\mu W_\nu^- W^{\nu+} + \partial_\nu W_\mu^- W^{\nu+}) + \frac{1}{2}(\partial_\mu Z'_\nu - \partial_\nu Z'_\mu) \\ &\quad \times (W^{\mu+} W^{\nu-} - W^{\mu-} W^{\nu+})]. \end{aligned} \quad (6)$$

The Majorana mass of  $N$  is thought to arise from the piece

$$L_{R\bar{R}\Delta} = fR^T C \epsilon(\tau/\sqrt{2}) \cdot \Delta R + \text{h.c.}, \quad (7)$$

where  $R^T$  is the  $SU(2)_R$  doublet  $(N, e_R)$ ,  $C$  is the charge conjugation matrix and  $\epsilon = i\tau_2$ .  $(\tau/\sqrt{2}) \cdot \Delta$  is the matrix

$$\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix},$$

and  $\epsilon(\tau/\sqrt{2}) \cdot \Delta$  is symmetric. This term gives a Majorana mass for the  $N$  neutrinos,  $M = fv_R/\sqrt{2}$ , so that  $g/f = M_W/M = r_W$ , and we have the interaction

$$L_{N\bar{N}H} = (g/\sqrt{2}r_W)\bar{N}^C H N. \quad (8)$$

$N^C$  is the field conjugate to  $N$ . It has been pointed out [19, 23] that the effective one-loop mass matrix for multi-generation unstable Majorana neutrinos is not Hermitian and, strictly speaking,  $N \neq N^C$ . But, here, at the level of tree-order calculations for a single generation,  $N = N^C$  will be assumed.

### 2.2 $\langle \sigma|v| \rangle$ for $N\bar{N} \rightarrow W_R^+ W_R^-$

Three amplitudes  $iM_{Z'}, iM_e, iM_H$  have been considered for  $N\bar{N} \rightarrow W^+W^-$ ; these arising, respectively, from  $N\bar{N} \xrightarrow{Z'} W^+W^-$  in the  $s$ -channel,  $N\bar{N} \xrightarrow{e} W^+W^-$  in the  $t$ -channel, and  $N\bar{N} \xrightarrow{H} W^+W^-$  in the  $s$ -channel. Earlier calculations, with  $r_W \ll 1$  [9], considered a heavy charged lepton exchange in the  $t$ -channel. In this paper, as required by the L–R model, an ordinary electron is considered to be exchanged.

$iM_{Z'}$  is calculated from (4) and (6),  $iM_e$  from (5), and  $iM_H$  from (3) and (8). One finds that, in the limit  $M \rightarrow 0$  and  $s \gg M_W^2$ ,  $iM_e$  and  $iM_{Z'}$  cancel in tree-order, as expected. With a massive  $M \gg M_W$ , an extra  $M$ -dependent term remains in  $iM_{Z'} + iM_e$ . On calculating  $\sum |\mathcal{M}|^2$ , it is found that the interference terms between  $iM_H$  and  $iM_{Z'} + iM_e$  cancel, and for  $r_W \ll 1$ , in the CM frame,

$$\begin{aligned} \sum |\mathcal{M}|^2 &= \frac{2g^4}{r_W^4} \left[ \frac{1}{t^2} E^2 \mathbf{k}^2 \sin^2 \theta_{\text{CM}} + \frac{16}{M^2(s - M_H^2)^2} E^6 \right]. \end{aligned} \quad (9)$$

$(E, \mathbf{k})$  is the 4-momentum of the  $N$  neutrino in the CM frame, and  $\theta_{\text{CM}}$  the angle of scattering.  $\mathbf{k}$  has been written for  $|\mathbf{k}|$ .

The thermally averaged cross-section times relative velocity is

$$\begin{aligned} \langle \sigma|v| \rangle &= (1/n_{\text{eq}}^2) \int d\pi_N d\pi_{\bar{N}} d\pi_{W^+} d\pi_{W^-} \\ &\quad \times (2\pi^4) \delta^4(p_N + p_{\bar{N}} - p_{W^+} - p_{W^-}) \\ &\quad \times \sum |\bar{\mathcal{M}}|^2 e^{-E_N/T} e^{-E_{\bar{N}}/T}. \end{aligned} \quad (10)$$

$n_{\text{eq}}$  is the equilibrium value of the number density  $n$  of the  $N$  neutrinos:

$$n_{\text{eq}} = g_N \left[ \frac{MT}{(2\pi)} \right]^{3/2} e^{-M/T}. \quad (11)$$

$g_N = 2$  for Majorana neutrinos. The measure  $d\pi_i = g_i d^3p_i / [(2\pi)^3 2E_i]$ .  $\sum |\bar{\mathcal{M}}|^2$  is the spin-averaged matrix element squared, with symmetry factor  $1/2!$  arising from the identification  $N = N^C$ .

The invariant integral  $\int d\pi_{W^+} d\pi_{W^-} (2\pi)^4 \delta^4(p_N + p_{\bar{N}} - p_{W^+} - p_{W^-}) \sum |\bar{\mathcal{M}}|^2$  is calculated in the CM frame, then transformed to the comoving “lab” frame, in which  $N, \bar{N}$  have energies  $E_N, E_{\bar{N}}$ , respectively, and thermally averaged according to (10). In all calculations,  $N$  neutrinos are non-relativistic, viz.  $E_N = M + \mathbf{k}_N^2/(2M)$ ,  $\mathbf{k}_N^2 \ll M^2$ . This is because the interesting region for decoupling studies of a massive particle has  $T \ll M$ .

The result is

$$\langle \sigma | v \rangle = \frac{1}{2!} \frac{g^4}{64\pi r_W^4} \left[ \frac{T}{M^3} + \frac{16}{M^2(4 - r_H^2)^2} \times \left\{ 1 - \frac{3T(4 + r_H^2)}{2M(4 - r_H^2)} \right\} \right]. \quad (12)$$

We have written  $r_H = M_H/M$ .

The first term arises from  $Z'$ - and  $e$ -exchange, and apart from the  $1/2!$  factor, agrees with the result of [11], where the results of [9] have been considered in the limit  $s \rightarrow 4M^2$ . The second term originates from  $H$ -exchange. In the calculation of this term, a further approximation has been made, viz.  $4\mathbf{k}^2 \ll |4M^2 - M_H^2|$ , i.e., this calculation is reliable provided  $M_H$  is not very close in value to  $2M$ . In [9], the  $H$ -exchange contribution was found to be negligible as  $s \rightarrow 4M^2$ , because of a factor  $(s - 4M^2)$  which arises for Dirac neutrinos. For Majorana neutrinos, this factor is absent, and this term cannot be neglected. As noted earlier, there is no interference between the  $H$ -exchange amplitude and those arising from  $Z'$ - and  $e$ -exchange. In [12], only the first term in (12) was considered, which is the approximation  $M_H \gg M$ .

The processes  $N\bar{N} \rightarrow F\bar{F}$ , where  $F$  is a relevant fermion, have not been considered here. For  $M \gg M_W$ , the contribution of these processes is small compared to that of  $N\bar{N} \rightarrow W^+W^-$  [9], and their effect on decoupling is overshadowed by the effects of  $N\bar{N} \rightarrow W^+W^-$  and the anomalous reduction of  $N$  neutrinos [12].

### 2.3 Thermally averaged decay width of $N$ neutrinos

The decay width for  $N \rightarrow W^+ + e^-$  can be calculated from (5). Calculation gives the spin-averaged matrix element squared in the neutrino rest frame

$$\sum |\bar{\mathcal{M}}|^2 = g^2 \left[ \frac{1}{2} M \mathbf{p} + \frac{M(M - \mathbf{p})M\mathbf{p}}{M_W^2} \right],$$

with  $\mathbf{p} = (1/2)M(1 - r_W^2)$  being the momentum of the decay products, resulting in the width

$$\Gamma_e = \frac{g^2 M}{32\pi} (1 - r_W^2)^2 \left( 1 + \frac{1}{2r_W^2} \right). \quad (13)$$

In the frame in which the neutrino has energy  $E_N$ , the width becomes

$$\Gamma_e^E = (M/E_N)\Gamma_e. \quad (14)$$

The thermally averaged decay width [24, 25] is defined as

$$\begin{aligned} \bar{\Gamma}_e &= (1/n_{\text{eq}}) \int d\pi_N d\pi_{W^+} d\pi_{W^-} \\ &\times (2\pi)^4 \delta^4(p_N - p_{W^+} - p_{W^-}) \sum |\bar{\mathcal{M}}|^2 e^{-E_N/T} \\ &= (1/n_{\text{eq}}) \int (g_N d^3p_N / (2\pi)^3) \Gamma_e^E e^{-E_N/T}. \end{aligned}$$

Using (13) and (14), one gets

$$\bar{\Gamma}_e = \frac{g^2 M}{32\pi} (1 - r_W^2)^2 \left( 1 + \frac{1}{2r_W^2} \right) \left( 1 - \frac{3T}{2M} \right). \quad (15)$$

The calculation has been done in the approximation  $T \ll M, \mathbf{k}_N^2 \ll M^2$ . For  $M \gg M_W^2$ ,

$$\bar{\Gamma}_e = \frac{g^2 M}{64\pi r_W^2} \left( 1 - \frac{3T}{2M} \right). \quad (16)$$

If the mass of the  $N$  neutrino is greater than the  $\Delta^+$  mass, (7) allows the decay  $N \rightarrow \Delta^+ + e^-$ . Correspondingly,

$$\sum |\bar{\mathcal{M}}|^2 = (1/2) f^2 (M^2 - M_+^2),$$

where  $M_+$  is the  $\Delta^+$  mass. In the rest frame of the neutrino, this decay width thus turns out to be

$$\Gamma_+ = (g^2/64\pi) (M^2/M_W^2) M (1 - r_+^2)^2, \quad (17)$$

where  $r_+ = M_+/M$ . There is another Yukawa piece of the Lagrangian:

$$L_{LR\Phi} = \sum_{i,j} (h_{ij} \bar{L}_i \Phi R_j + h'_{ij} \bar{L}_i \tau_2 \Phi^* \tau_2 R_j) + \text{h.c.} \quad (18)$$

The scalar field  $\Phi$  transforms under the gauge group as  $(1/2, 1/2, 0)$  and is represented by the matrix

$$\Phi = \begin{pmatrix} \phi^0 & \phi'^+ \\ \phi^- & \phi'^0 \end{pmatrix}.$$

Such pieces have been used in different models to break  $CP$  and induce baryogenesis through leptogenesis [4, 5, 17–20]. If one generation is considered, the coupling constant  $h$  contributes to the electron mass and must be very small. To fit the observed baryon asymmetry, with  $N$  mass in the TeV range,  $|h_{ij}|^2$  values of the order of  $10^{-10}$  to  $10^{-13}$  have been considered [4, 5]. In this situation, as  $CP$  breaking has not been considered here, and only one generation having been taken into account, it has not been thought useful to consider  $N$ -decays arising from (18). In any case, it will be found that any enhancement of the decay width given in (15) and (16) will only strengthen the main conclusion of the paper.

### 2.4 Anomalous generation of Majorana neutrinos

The one generation, sphaleron-mediated, fermion number violating transition rate per unit volume, with  $|\Delta L| = 1$ ,  $|\Delta B| = 1$ , for the quantum anomaly, involving the  $SU(2)_R$

gauge group, was written, in [12], by extrapolation from the  $SU(2)_L$  case [26], as

$$A_R = (1.4 \times 10^6) \left( \frac{bM_W^7}{g^6 T^3} \right) \left[ 1 - \left( \frac{T}{zM_W} \right)^2 \right]^{7/2} \times \exp \left\{ -\frac{16\pi M_W}{g^2 T} \left[ 1 - \left( \frac{T}{zM_W} \right)^2 \right]^{1/2} \right\}. \quad (19)$$

$M_W$  is the zero temperature  $W_R$  mass.  $z = T_R/M_W$ , where  $T_R$  is the critical temperature associated with the breaking of the  $SU(2)_R$  gauge symmetry. So,  $z$  is essentially a quantity which reflects the uncertainty in the values of the L–R model parameters, while  $b$  captures the uncertainties involved in the extrapolation from the  $SU(2)_L$  to the  $SU(2)_R$  case in addition to those in the estimation of the prefactor of the anomaly driven transition [27].

In [12], the anomalous rate of reduction of  $N$  neutrinos was considered, maintaining a distinction between  $N$  and  $N^C$ . Here,  $N = N^C$  neutrinos are considered, to maintain uniformity with the  $N\bar{N} \rightarrow W^+W^-$  calculations. This entails an extra factor of 2, as seen below.

For an anomalous process  $l$ , with  $\Delta L = +1$ , such that

$$l : i + j + \dots \rightarrow N + a + b + \dots,$$

one writes [12]

$$A_l = \int d\pi_N d\pi_a d\pi_b \dots d\pi_i d\pi_j \dots |\mathcal{M}_l|^2 (2\pi)^4 \times \delta^4(p_N + p_a + p_b + \dots - p_i - p_j - \dots) f_N^{\text{eq}} f_a f_b \dots = I_l n_{\text{eq}}, \quad (20)$$

where  $I_l$  contains the result of the phase space integrations, apart from  $n_{\text{eq}}$  [24,25], and  $i, j, \dots, a, b, \dots$  are all supposed to be in equilibrium.

Taking the view that leptogenesis and baryogenesis are effects of a smaller order,  $CP$ -symmetry is assumed. Then [12] for each process  $l$ , there is a  $\Delta L = -1$  process

$$\bar{l} : N + a + b + \dots \rightarrow i + j + \dots,$$

with the same  $|\mathcal{M}_l|^2$ . For this process,

$$A_{\bar{l}} = \int d\pi_N d\pi_a d\pi_b \dots d\pi_i d\pi_j \dots |\mathcal{M}_l|^2 (2\pi)^4 \times \delta^4(p_N + p_a + p_b + \dots - p_i - p_j - \dots) f_N f_a f_b \dots = I_l n \quad (21)$$

The  $\Delta L = +1$  processes, generating  $N$  neutrinos, add up to

$$\sum_l A_l = n_{\text{eq}} \sum_l I_l = \frac{1}{2} A_R \exp(-\beta\mu_L/2),$$

from [26].  $\mu_L = \mu_N$  is the chemical potential ( $\mu_e = 0$ , as electrons are in equilibrium). The  $1/2$  factor arises from the assumption [12], that, to a first approximation, the rate of generation of one member of a lepton doublet may

be taken to be the same as that of the other, near equilibrium (not a bad condition at decoupling when the neutrinos are just falling out of equilibrium). Similarly,

$$\sum_{\bar{l}} A_{\bar{l}} = n \sum_l I_l = \frac{1}{2} A_R \exp(+\beta\mu_L/2).$$

So, for small  $\mu_N$  [26],

$$n_{\text{eq}} \sum_l I_l \approx \frac{1}{2} A_R (1 - \beta\mu_N/2), \\ n \sum_l I_l \approx \frac{1}{2} A_R (1 + \beta\mu_N/2).$$

One gets

$$\sum_l I_l = A_R / (n + n_{\text{eq}}),$$

and the anomalous rate of reduction of  $N$  neutrinos per unit volume becomes

$$A_N = \sum_{\bar{l}} A_{\bar{l}} - \sum_l A_l = \frac{n - n_{\text{eq}}}{n + n_{\text{eq}}} A_R. \quad (22)$$

This has an extra factor of 2, compared to [12], because anti-particle processes, which had to be considered separately there, do not appear here, because of the assumption  $N = N^C$ .

### 3 Effect of decay on decoupling

#### 3.1 The Boltzmann equation for $N$ neutrinos

Using the results of the last section, one can write the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -2\langle\sigma|v|\rangle(n^2 - n_{\text{eq}}^2) - 2\bar{\Gamma}_e(n - n_{\text{eq}}) - \frac{n - n_{\text{eq}}}{n + n_{\text{eq}}} A_R, \quad (23)$$

where the second term on the left gives the effect of the expansion, and the three terms on the right are to be taken from (12), (16), and (22), respectively. The expression for the Hubble parameter  $H$  is

$$H = 1.66g^{*(1/2)}T^2/M_{Pl}. \quad (24)$$

$g^*$  is taken  $\approx 100$ , and  $M_{Pl} = 1.22 \times 10^{19}$  GeV. The factor 2 with  $\langle\sigma|v|\rangle$  appears because two neutrinos are disappearing in  $N\bar{N} \rightarrow W^+W^-$ , considering  $N = N^C$ . The 2 factor with  $\bar{\Gamma}_e$  appears because of the two decay channels  $N \rightarrow W^+ + e^-$  and  $N \rightarrow W^- + e^+$ . Writing  $x = M/T$  and  $Y = n/s$ , where  $s = g^{*S}(2\pi^2/45)T^3$ , with  $g^{*S} \approx 100$ , (23) becomes

$$\frac{dY}{dx} = -f(x)(Y^2 - Y_{\text{eq}}^2) - d(x)(Y - Y_{\text{eq}}) - g(x) \frac{Y - Y_{\text{eq}}}{Y + Y_{\text{eq}}}. \quad (25)$$

In (25),

$$f(x) = \left( \frac{1.41 \times 10^{16} \text{ GeV}}{M_W} \right) \left( \frac{a}{x} \right)^3 \times \left[ \frac{2x}{(1 - r_H^2/4)^2} - h(r_H) \right], \quad (26)$$

where  $a = 1/r_W = M/M_W$  and

$$h(y) = \frac{1}{(1 - y^2/4)^3} \times [1 + y^2 \{ (1 - y^2/8)(1 - y^2/4) + 5/4 \}], \quad (27)$$

$$d(x) = \left( \frac{3.08 \times 10^{15} \text{ GeV}}{M_W} \right) ax \left[ 1 - \frac{3}{2x} \right], \quad (28)$$

$$g(x) = \left( \frac{3.06 \times 10^{23} \text{ GeV}}{M_W} \right) \frac{b}{a} \left[ \frac{1}{u^2} - \frac{1}{z^2} \right]^{7/2} \times \exp \left\{ -118.98 \left[ \frac{1}{u^2} - \frac{1}{z^2} \right]^{1/2} \right\}, \quad (29)$$

with  $u = a/x = T/M_W$ .

Writing  $Y = Y_{\text{eq}} + \Delta$ , it is noted that, before decoupling,  $Y$  is close to  $Y_{\text{eq}}$ , and  $\Delta$  may be put equal to zero. Then (25) may be put in the form

$$\Delta = \frac{-Y'_{\text{eq}}}{f(x)(2Y_{\text{eq}} + \Delta) + d(x) + \frac{g(x)}{2Y_{\text{eq}} + \Delta}}.$$

From  $n_{\text{eq}}$  in (11), one gets

$$Y_{\text{eq}} = 2.89 \times 10^{-3} x^{3/2} e^{-x}, \quad \text{and} \quad Y'_{\text{eq}} \approx -Y_{\text{eq}},$$

at decoupling, when it is expected that  $x = x_d \gg 1$ . The criterion for decoupling may be taken as  $\Delta = c' Y_{\text{eq}}$ , where  $c'$  is of order 1. As in [12],  $c'$  is chosen to be 1. Then the condition for decoupling is

$$3f(x_d)Y_{\text{eq}}(x_d) + d(x_d) + \frac{g(x_d)}{3Y_{\text{eq}}(x_d)} = 1. \quad (30)$$

This is the key condition in our analysis of decoupling. Each factor on the lhs of (30) is positive<sup>1</sup>. So, there is no possibility for cancellation between different terms. We will show in the following subsections that, in fact, the condition can never be satisfied. The argument proceeds as follows. First, we consider (30), excluding the second (decay) term in the lhs. We show that there is a value  $x_a$ , which we obtain numerically below, such that, for  $x < x_a$  or  $x > x_a$ , the lhs is, respectively, greater or less than unity in the absence of decay. We then check that for  $x > x_a$  the decay term is much larger than unity so that there is no value of  $x$  ( $= M/T$ ) for which (30) is satisfied.

<sup>1</sup> At first sight it might seem from (26) that the first term on the lhs of (30) is not always positive. That this is not the case can be seen by noting that (a) this analysis is valid only when  $r_H$  is away from the value 2, and, from (27), (b)  $h(r_H)$  is negative for  $r_H > 2$  and quickly assumes its asymptotic value of  $-2$  for  $r_H > 5$ , and (c) for  $r_H \leq 1$ , though  $h(r_H)$  is positive, the first term in the square brackets in (26) is dominant for  $x \gg 1$  (expected at decoupling)

**Table 1.**  $x = M/T$  at decoupling for different choices of  $a = M/M_W$  and  $r_H = M_H/M$  without the inclusion of the decay contribution in the Boltzmann equation.  $M_W$  has been chosen to be 4000 GeV

$r_H = 1$		$r_H = 3$		$r_H = 10$	
$a$	$x$	$a$	$x$	$a$	$x$
2	25.79	2	25.06	2	22.30
5	28.49	5	27.74	5	24.89
10	30.54	10	29.77	10	26.86
20	32.60	20	31.81	20	28.83
50	35.32	50	34.52	50	32.18
75	73.77	75	73.77	75	73.77
100	238.83	100	238.83	100	238.83

### 3.2 Decoupling in the absence of decay

First,  $d(x_d)$  is omitted, and the decoupling condition

$$l(x) = 3f(x)Y_{\text{eq}}(x) + \frac{g(x)}{3Y_{\text{eq}}(x)} = 1 \quad (31)$$

is solved to give  $x = x_a$ .  $x_a$  represents the value of  $M/T$  for which decoupling would occur in the absence of decay.

$Y_{\text{eq}}$  gives a factor  $e^{-x}$ , and so the term with  $f(x)$  increases as  $x$  decreases.  $g(x)$  has an exponential factor of the form  $e^{-E_{\text{sp}}/T} = e^{-Kx/a}$  [12, 14, 26, 28, 29], where  $E_{\text{sp}}$  is the energy of the sphaleron mode which decays to cause anomalous  $L$  generation. The kinematic constraint on  $N$  production,  $E_{\text{sp}} > M$ , gives  $K/a > 1$  [12] so  $g(x)/Y_{\text{eq}} \sim e^{-((K/a)-1)x}$  and this again increases as  $x$  decreases. This means that  $l(x) > 1$  if  $x < x_a$ . So,  $x < x_a$  will not satisfy the decoupling condition (30). Therefore, on the whole, it may be said approximately [24] that, for  $x > x_a$ , the annihilation rate plus the anomalous reduction rate is less than  $H$ .

This programme is followed numerically. First, for definiteness, we fix  $b$  and  $z$ , the two parameters in the expression for  $g(x)$  – see (29) – at the values 1 and 4, respectively [12].  $a$  is varied from 2–100, for<sup>2</sup>  $r_H = 0, 1, 3, 10$  and  $x_a$  found by solving (31) numerically.

The results for  $M_W = 4000$  GeV are shown in Table 1 for  $r_H = 1, 3, 10$ . There is a clear trend, showing an increase in  $x_a$  for an increase in  $a$  (constant  $r_H$ ), and a decrease in  $x_a$  for an increase in  $r_H$  (constant  $a$ ). The values for  $r_H = 100$  and  $r_H = 1000$ , we have checked, do not differ at all. It is safe to say that  $x_a > 20$  for the parameter ranges considered.

We now address the uncertainty regarding the anomalous rate [27]. The uncertainty is embodied in the parameters  $b$  and  $z$  appearing in the expression for this rate. The parameter  $z = T_R/M_W$  is of order unity ( $z = 3.8$  in the  $SU(2)_L$  case [26]). To gauge the sensitivity of the results on  $z$ , a calculation was made with  $a = 50$  (to give sufficient weight to the anomalous transition factor  $g(x)$ ) and  $r_H = 1$ .  $x_a$ , obtained by solving (31), was 42 for  $z = 2$ , 35.6 for  $z = 3$ , and reached an asymptotic value of 35.3 for  $z \geq 4$ . For  $r_H = 10$ ,  $x_a$  was 42 for  $z = 2$ , 35 for  $z = 3$ , and it had an asymptotic value of 31.5. The conclusion

<sup>2</sup>  $h(r_H)$  assumes its asymptotic value of  $-2$  for  $r_H > 5$

**Table 2.** Effect of uncertainty in anomalous rate on the results of Table 1

$a = 2$		$a = 50$	
$b$	$x$	$b$	$x$
$10^3$	22.30	$10^3$	36.79
$10^2$	22.30	$10^2$	35.11
10	22.30	10	33.50
1	22.30	1	32.18
$10^{-1}$	22.30	$10^{-1}$	31.60
$10^{-2}$	22.30	$10^{-2}$	31.47
$10^{-3}$	22.30	$10^{-3}$	31.46

**Table 3.** Same as in Table 1 but for  $M_W = 2000$  GeV

$r_H = 1$		$r_H = 3$		$r_H = 10$	
$a$	$x$	$a$	$x$	$a$	$x$
2	26.47	2	25.73	2	22.95
5	29.18	5	28.41	5	25.54
10	31.23	10	30.45	10	27.51
20	33.28	20	32.49	20	29.49
50	36.00	50	35.20	50	32.75
75	75.01	75	75.01	75	75.01
100	242.83	100	242.83	100	242.83

$x_a > 20$  of the last paragraph is not disturbed. In [12], the uncertainty in  $b$  was taken into account by varying it through six orders of magnitude about  $b = 1$ . We do a similar calculation for  $z = 4$ ,  $r_H = 10$ ,  $M_W = 4000$  GeV and present the results in Table 2. It is clear that even this large variation of  $b$  through six orders of magnitude does not affect the conclusion of the previous paragraph.

For comparison, the results for  $M_W = 2000$  GeV, for the same values of the other parameters, are shown in Table 3.

The expectation that  $l(x)$  – see (31) – decreases below 1, as  $x$  increases through  $x_a$ , was verified numerically for  $M_W = 4000$  GeV, and  $a = 10, 20, 50, 75$ , for each of the values  $r_H = 1, 10$ . Table 4 shows the results for  $r_H = 1, 10$ , and  $a = 10, 50$ .

### 3.3 Effect of decay

From the numerical results of the previous subsection it would be safe to say that for  $x < 20$ ,  $l(x) > 1$ . Now, if one looks at  $d(x)$ , see (28), it is clear that, for  $x > 20$ ,  $d(x) \gg 1$ , and, moreover, as  $x$  increases further,  $d(x)$  increases. So, there is no possibility of the decoupling condition (30) being satisfied.

$d(x)$ , of course, has a simple physical meaning. In the lhs of (23),

$$\frac{dn}{dt} + 3Hn = Hsx \frac{dY}{dx}.$$

Comparing (23) and (25),  $d(x) = (2\bar{\Gamma}_e/Hx)$ . If  $d(x) \gg 1$  for  $x = x_a$ , with  $x_a \sim 20$ , this means that  $\bar{\Gamma}_e \gg H$  at this point. Considering the physical meaning of  $x_a$ , it may be concluded that although at temperatures lower than  $T_a = M/x_a$ , the annihilation rate plus anomalous

reduction rate falls below the expansion rate, the decay rate remains much faster than the expansion, and this prevents decoupling.

In the absence of decoupling, the fast decay constrains the  $N$  neutrino number density to follow the equilibrium density  $\sim e^{-M/T}$ . There is no out-of-equilibrium decay.

One may check that this conclusion is not an artefact of the approximation  $M^2 \gg M_W^2$  in the calculations. Without this approximation, (15) gives the decay rate

$$\bar{\Gamma}_e = \frac{g^2 M (a^2 - 1)^2 (a^2 + 2) (1 - (3/2x))}{64\pi a^4}, \quad (32)$$

so that

$$d(x) = \left( \frac{3.08 \times 10^{15} \text{ GeV}}{M_W} \right) \times \frac{(a^2 - 1)^2 (a^2 + 2)x}{a^5} \left[ 1 - \frac{3}{2x} \right]. \quad (33)$$

A simple calculation shows that if  $d(x)$  is to be  $< 1$ , for  $x \sim 20$ , then one must have  $a \sim 1$ , to 1 part in  $10^7$  (for  $M_W = 4000$  GeV). There is no reason for such a fine tuning between the  $N$  and  $W$  masses.

If the  $N$  decay width is augmented by the  $\Delta^+$ ,  $e^-$  channel, i.e., if  $M > M_+$ ,

$$d(x) = \left( \frac{3.08 \times 10^{15} \text{ GeV}}{M_W} \right) \left[ \frac{(a^2 - 1)^2 (a^2 + 2)}{a^5} + a(1 - r_+^2) \right] \times x \left[ 1 - \frac{3}{2x} \right]. \quad (34)$$

In this case, if  $d(x)$  is to be  $< 1$  for  $x \sim 20$ , with  $M_W = 4000$  GeV, not only must  $r_W \sim 1$ , but also  $r_+ \sim 1$ , each to 1 part in  $10^7$ ; an unacceptable situation.

Clearly, if the decay width is further augmented by introducing other channels, this will change  $\Gamma_e$ , but not the  $x$ -dependence of  $\bar{\Gamma}_e$  or  $d(x)$ , so that  $d(x) < 1$  for  $x \sim 20$  will be an even remoter possibility.

$d(x)$  can be decreased by increasing  $M_W$ . Taking  $r_H = 1$ , a little calculation shows that  $d(x) = 0.999$  and  $l(x) = 0.001$  for  $x = 4.25$  and  $M_W = 1.7 \times 10^{16}$  GeV if  $a = 2$ , and for  $x = 6.6$  and  $M_W = 1.6 \times 10^{17}$  GeV if  $a = 10$ .

This scale of  $M_W$  agrees with [20] where the decoupling condition was, however, chosen simply as  $\bar{\Gamma}_e < H$  at  $T = M$ ,  $\bar{\Gamma}_e$  having been assigned the value  $g^2 T/8\pi$ . In this paper, we have calculated

$$\Gamma_e = \frac{g^2 (M^2 - M_W^2)^2 (1 + M^2/2M_W^2)}{32\pi M^3},$$

and taken the thermal average of  $(M/E_N)\Gamma_e$  for  $T < M$ . We have used a decoupling criterion which includes the effect of annihilations and anomalous reduction, for a wide range of the parameters  $r_H = M_H/M$  and  $r_W = M_W/M$ . Also, our method of obtaining an approximate solution of the Boltzmann equation keeps the temperature of decoupling open and calculable, e.g., in the previous paragraph it was found that decoupling occurs for  $M > 10^{16}$ , ( $10^{17}$ ) GeV at  $T = M/4$  ( $M/6$ ), and not at  $T = M$ .

**Table 4.** Behavior of  $l(x)$  (31) around  $x_a$  in the absence of decay.  $M_W = 4000$  GeV for this table

$r_H = 1$				$r_H = 10$			
$a = 10$		$a = 50$		$a = 10$		$a = 50$	
$x$	$l(x)$	$x$	$l(x)$	$x$	$l(x)$	$x$	$l(x)$
20	$45.23 \times 10^3$	30	227.87	20	$14.61 \times 10^2$	25	$13.13 \times 10^3$
35	$1.09 \times 10^{-2}$	40	$8.70 \times 10^{-3}$	30	$3.67 \times 10^{-2}$	40	$1.51 \times 10^{-4}$

So, the main point which emerges is that for there to be decoupling of massive neutrinos in the L–R model, it is necessary to consider right-handed gauge boson mass values far above the physically expected L–R mass scale. Further, as  $M = aM_W$ , the see-saw mechanism will, then, give values of the  $\nu$  mass, which will be unacceptably small, when compared to neutrino oscillation values of  $\Delta m^2$ .

## 4 Conclusions

It is to be concluded that, in the L–R model, with right-handed neutrino mass greater than the  $W_R$  mass, even when the annihilation plus anomalous reduction rate for these neutrinos has fallen below the expansion rate, the decay remains faster than expansion, and becomes increasingly faster. So, massive right-handed neutrinos will decay while remaining in equilibrium.

We find that this is true for a wide range of values of  $M_W/M$  and  $M_H/M$ , and also for a wide allowance of the uncertainty in the anomalous rate.

As the equilibrium number density varies as  $e^{-M/T}$ , right-handed neutrinos rapidly dwindle in number once the temperature falls below their mass. The decay products, with much lower masses, equilibriate immediately. Hence, there is no question of influencing the present density of the universe, CMBR, and nucleosynthesis or later events, and no mass bound can be set for right-handed neutrinos in the L–R model, if their mass is greater than the  $W_R$  boson mass (apart from an upper bound from unitarity [10, 11]).

Because there is no out-of-equilibrium decay, the lepton (and baryon) number generation scenarios, utilising the decay of massive Majorana neutrinos will not work in the L–R model if the neutrino mass is greater than the  $W_R$  boson mass, a result also previously noted in [20].

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